

# Controller Tuning Method for Non-Linear Conical Tank System

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## ABSTRACT

In this paper, we propose a new technique for implementing optimum controller for a conical tank. The objective of the controller is to maintain the level inside the process tank in a desired value. Hence an attempt is made in this paper as Internal Model Based PID controller design for conical tank level control. For each stable operating point, a first order process model was identified using process reaction curve method. The real time implementation is done in Simulink using MATLAB. The experimental results shows that proposed control scheme have good set point tracking and disturbance rejection capability.

Index Terms: Mathematical model, Final control element, PID Controller, IMC and MATLAB Simulink.

## 1. INTRODUCTION

Each and every industry phase the flow control and level control problem. So that canonical tank level process is used. Canonical tank level process is used. Canonical tank is the highly non-linear system. Because of this, reason sometimes output may be affected. Due to its shape, the conical tank is lead to non-linearity control of conical tank is the challenging problem so many researchers have been carried out in the level control of conical process. For complete drainage of fluids, a conical bottomed cylindrical tank is used in some of the process industries, where its nonlinearity might be at the bottom only. The drainage efficiency can be improved further if the tank is fully conical. But continuous variation in the tank system makes it highly non-linear and hence the liquid level control in such systems is difficult. A conical shaped tank system are mainly used in Colloidal mills, Leaching extractions in pharmaceutical and chemical industries, food processing industries, Petroleum industries, Molasses, Liquid feed and Liquid fertilizer storage, Chemical holding & mix tank, Biodiesel processing and reactor tank. To avoid settlement and sludge in Storage and holding tanks, the conical tanks are used.

## 2. PROPOSED WORK

### 2.1 Experimental Setup

The level process station was used to conduct the experiments and collect the data. The computer acts as a controller. It consists of the software used to control the level process station. The setup consists of a process tank, reservoir tank, control valve, I to P converter, level sensor and pneumatic signals from the compressor. When the setup is switched on, level sensor senses the actual level values initially then signal is converted to current signal in the range 4 to 20mA. This signal is then given computer through data acquisition cord. Based on the values entered in the controller Settings and the set point the computer will take control action the signal sent

by the computer is taken to the station again through the cord. This signal is then converted to pressure signal using I to P converter. Then the pressure signal acts on a control valve which controls the flow of water in to the tank there by controlling the level.

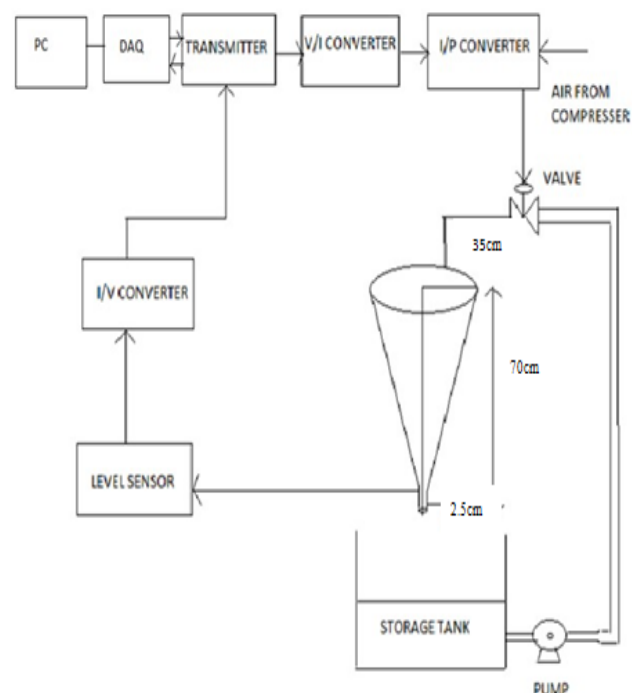


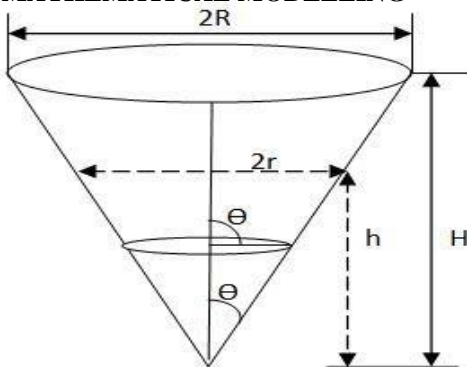
Fig.1. Level Control of Conical Tank System

### 2.2 Description of the Conical Tank Level Process

The tank is made up of stainless steel body and is mounted over a stand vertically. Water enters the tank from the top and leaves the bottom to the storage tank. The System specifications of the tank are as follows,

**Technical Specification**

Equipments	Details
Conical tank	Stainless steel body, height– 70 cm, Top diameter–35 cm Bottom diameter 2.5cm
Differential Pressure Level Transmitter	Differential Pressure Level Transmitter
Pump	Centrifugal 800LPH
Control Valve	Size 1/4 Pneumatic actuated Type: Air to open, Input 3-15PSI
Rota meter	Range 0-600 LPH

**3. MATHEMATICAL MODELLING**

Where,

- R=Radius of the tank [Constant]
- H=Total height [Constant]
- r=radius of the liquid level [Constant]
- h=level of the water [variable]

Consider the tank with the angle  $\theta$ .

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{r}{h} = \frac{R}{H}$$

$$r/h = R/H$$

$$r = R \cdot h/H \text{ ----->1}$$

$$\text{Area} = \pi r^2 \text{ ----->2}$$

Sub 1 in 2 and differentiating,

$$\frac{da}{dt} = \pi \left[ \frac{R}{H} \right]^2 \cdot 2h \frac{dh}{dt} \text{ ----->3}$$

$$\text{Volume of cone, } V = \frac{1}{3} \pi r^2 h \text{ ----->4}$$

Differentiating the above equation

$$\frac{dv}{dt} = \frac{1}{3} \left[ A \frac{dh}{dt} + \frac{da}{dt} h \right] \text{ ----->5}$$

Sub 3 in 5

$$\frac{dv}{dt} = \frac{1}{3} \frac{dh}{dt} \left[ A + 2h^2 \pi \left[ \frac{R}{H} \right]^2 \right] \text{ ----->6}$$

By Newton's law:

$$F_{in} - F_{out} = \frac{1}{3} \frac{dh}{dt} \left[ A + 2h^2 \pi \left[ \frac{R}{H} \right]^2 \right] \text{ ----->7}$$

$$F_{out} = k \sqrt{h} \text{ ----->8}$$

$$F_{in} - k \sqrt{h} = \frac{1}{3} \frac{dh}{dt} \left[ A + 2h^2 \pi \left[ \frac{R}{H} \right]^2 \right]$$

$$\frac{dh}{dt} = \frac{3[F_{in} - k\sqrt{h}]}{A + 2h^2 \pi \left[ \frac{R}{H} \right]^2}$$

Sub Area in above equation,

$$\frac{dh}{dt} = \frac{3[F_{in} - k\sqrt{h}]}{\pi \left[ \frac{R}{H} \right]^2 + 2h^2 \pi \left[ \frac{R}{H} \right]^2}$$

$$= \frac{F_{in}}{\pi h^2 \left[ \frac{R}{H} \right]^2} - \frac{k h^{1/2}}{\pi h^2 \left[ \frac{R}{H} \right]^2}$$

$$\text{Let, } \alpha = \frac{1}{\pi h^2 \left[ \frac{R}{H} \right]^2} = \beta = k\alpha$$

$$= \frac{\alpha F_{in}}{h^2} - \beta \frac{h^{1/2}}{h^2}$$

$$\frac{dh}{dt} = \alpha F_{in} h^{-2} - \beta h^{-3/2} \text{ ----->9}$$

Taylor Series,

For linearising  $Fh^{-2}$  And  $h^{-3/2}$  in ----->9

$$F(h_1 F_{in}) = F(h_s F_{in_s}) - 2F_{in_s} h_s^{-3} (h - h_s) + h_s^{-2} (F - F_{in_s}) \text{ ---->10}$$

$$\frac{dh}{dt} = h^{-3/2} [\text{Initially}]$$

$$h^{-3/2} = h_s^{-3/2} - \frac{3}{2} h_s^{-5/2} (h - h_s) \text{ ----->11}$$

Sub 11 and 10 in 9

$$\frac{dh}{dt} = \alpha [F(h_s, F_{in_s}) - 2F_{in_s} h_s^{-3} (h - h_s) + h_s^{-2} (F - F_{in_s})] - \beta [h_s^{-3/2} - \frac{3}{2} h_s^{-5/2} (h - h_s)] \text{ ----->12}$$

At initial and steady state condition,

$$\frac{d(h-h_s)}{dt} = \alpha [-2F_{in_s} h_s^{-3} (h - h_s) + h_s^{-2} (F - F_{in_s}) + \frac{3}{2} \beta h_s^{-5/2} (h - h_s)] \text{ ----->13}$$

Let  $y = (h - h_s)$  and  $u = F - F_{in_s}$  sub in equ ---->13

$$\frac{dy}{dt} = -2\alpha F_{in_s}^{-3} y + \alpha h_s^{-2} u + \frac{3}{2} \beta h_s^{-5/2} y \text{ ----->14}$$

$$\frac{dy}{dt} = -2\beta h_s^{1/2} h_s^{-3} y + \alpha h_s^{-2} u + \frac{3}{2} \beta h_s^{-5/2} y \text{ ----->15}$$

$$\frac{dy}{dt} = -\left(\frac{1}{2}\right) \beta h_s^{-5/2} y + \alpha h_s^{-2} u \text{ ----->16}$$

$$\left[ \frac{2}{\beta} h_s^{-2} \right] \frac{dy}{dt} + y = \alpha h_s^{-2} u \left[ \frac{2}{\beta} h_s^{5/2} \right] \text{ ----->17}$$

$$\tau \frac{dy}{dt} + y = c u \text{ ----->18}$$

Comparing 17 and 18,

$$\tau = \frac{2}{\beta} h_s^{5/2} \quad C = 2 \frac{\alpha}{\beta} h_s^{1/2}$$

Applying laplace transform for equ ---->18

$$\tau s Y(s) + Y(s) = C U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{C}{\tau s + 1}$$

The transfer function for the different height of the tank,

Model	Height	Transfer function
1	10	$\frac{Y(s)}{U(s)} = \frac{3.16}{62.08s + 1}$
2	15	$\frac{Y(s)}{U(s)} = \frac{3.87}{170.66s + 1}$
3	20	$\frac{Y(s)}{U(s)} = \frac{4.47}{350.44s + 1}$

The transfer function of the model is,

$$GV(s) = \frac{0.13}{3s + 1}$$

The transfer of the sensor is,

$$H(s) = \frac{1}{10s + 1}$$

The process transfer function is represented as,

Model	Height	Transfer function
1	10	$\frac{y(s)}{U(s)} = \frac{0.4108}{186.24s^2 + 65.08s + 1}$
2	15	$\frac{y(s)}{U(s)} = \frac{0.5031}{511.98s^2 + 173.66s + 1}$
3	20	$\frac{y(s)}{U(s)} = \frac{0.5811}{1051.32s^2 + 353.44s + 1}$

The open loop response of the system is,

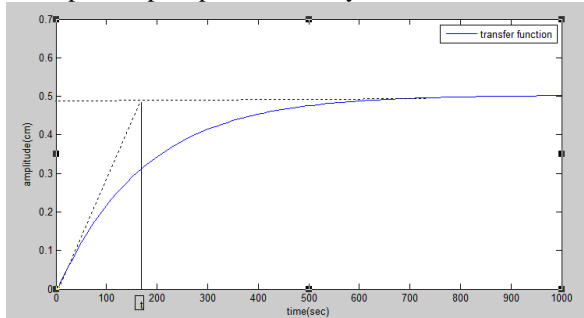


Fig. Response of the Open Loop System

#### 4. PID CONTROLLER

A proportional-integral-derivative controller (PID controller) is a generic control loop feedback mechanism widely used in many control systems. A PID controller calculates an error value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error by adjusting the process control inputs. The PID controller is simple and robust and hence widely used in most of the process industries. The controller parameters can be tuned using Cohen & Coon method.

##### Cohen and Coon Method

Cohen and Coon method is commonly referred to as open loop response method for tuning the parameters of conventional controllers. Once the open loop response is obtained the required values are noted. In order to obtain the required gain values of the PID controller the following tabulation can be used

Table 1: Cohen and Coon tuning formulae

Type	$K_p$	$T_i$	$T_d$
P	$\left(\frac{\tau}{Kt_d}\right)\left(1 + \frac{t_d}{3\tau}\right)$	-	-
PI	$\left(\frac{\tau}{Kt_d}\right)\left(0.9 + \frac{t_d}{12\tau}\right)$	$t_d \left(\frac{30 + \frac{3t_d}{\tau}}{9 + \frac{20t_d}{\tau}}\right)$	-
PID	$\left(\frac{\tau}{Kt_d}\right)\left(\frac{4}{3} + \frac{t_d}{4\tau}\right)$	$t_d \left(\frac{32 + \frac{6t_d}{\tau}}{13 + \frac{8t_d}{\tau}}\right)$	$t_d \left(\frac{4}{11 + \frac{2t_d}{\tau}}\right)$

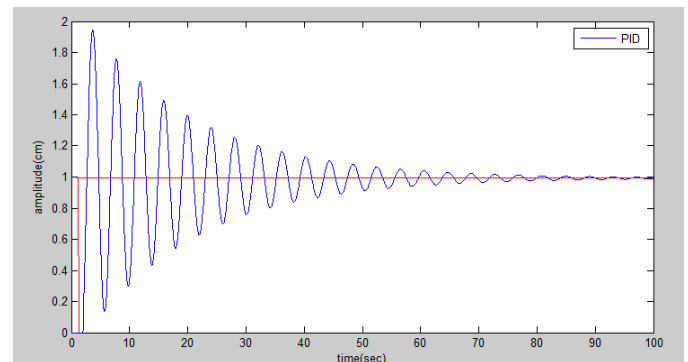
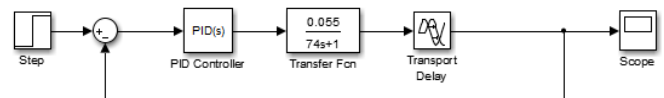
Using the above formulae the gain values can be obtained.

Model	Height	Transfer function	K p	Ti	Td
1	10	$\frac{0.055}{74s+1} e^{-1s}$	1973	2.44	0.36
2	15	$\frac{0.172}{219s+1} e^{-1s}$	1697	2.45	0.363
3	20	$\frac{0.224}{349s+1} e^{-1s}$	1713	2.45	0.362

The proportional control will make the system to be stable but there exists an offset error. But increasing the proportional gain value makes the system unstable and the response oscillates continuously. Integral control is used to eliminate the offset. It corrects the offset overtime by shifting the proportional band. If the reset time is too small, the response oscillates continuously. If the reset time is too long, the system will take long time to settle out. Derivate action is to make the process variable to settle quickly after the disturbance. This can be mainly used for slow process. The derivative control also reduces the peak overshoot.

##### Block Diagram of a PID Controller

Therefore, Proportional Integral Derivative (PID) controller is used to obtain the closed loop response of the conical tank system.

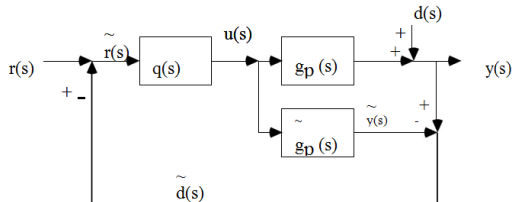


Response of conventional PID

#### 5. IMC BASED PID CONTROLLER

The ability of proportional-integral (PI) and Proportional-Integral-Derivative (PID) controllers to meet most of the control objectives has led to their widespread acceptance in the control industry. It is because, for practical applications or an actual process in industries PID controller algorithm is simple and robust to handle the model inaccuracies. This error becomes severe for the process with time delay. For this we have taken some transfer functions with significant time delay. model based control systems are often used to track set points and reject low disturbances. The IMC design

procedure is exactly the same as the open loop control design procedure. The IMC filter tuning parameter “ $\lambda$ ” is used to avoid the effect of model uncertainty. The normal IMC design procedure focuses on set point responses but with good set point responses good disturbance rejection is not assured, especially those occurring at the process inputs. A modification in the design procedure is proposed to enhance input disturbance rejection and to make the controller internally stable.



IMC structure

Standard Feedback Diagram Illustrating the Equivalence with Internal Model Control. The feedback controller,  $g_c(s)$ , contains both the internal model,  $\tilde{g}_p(s)$ , and internal model controller,  $q(s)$ .

The standard feedback controller is a function of the internal model  $g_p(s)$  and internal model controller  $q(s)$  shown in equation below.

$$g_c(s) = \frac{q(s)}{1 - \tilde{g}_p(s)q(s)}$$

The open loop transfer function of the process is

$$G_m(s) = \frac{0.055}{74s+1} e^{-1s} \quad \text{-----1}$$

The IMC controller transfer function is

$$G_{imc}(s) = \frac{1}{G_m(s)} f(s) \quad \text{-----2}$$

The best way to select the closed loop servo transfer function  $f(s)$  to make physically realizable is

$$f(s) = \frac{1}{\lambda_c(s)+1}$$

Let  $\lambda_c(s)=1$  in above eqn,

$$f(s) = \frac{1}{5s+1} \quad \text{-----3}$$

Sub 1 and 3 in 2

$$G_c(s) = \frac{74s}{0.055s+1}$$

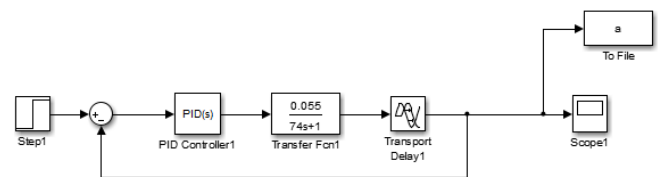
It is compared with conventional PID as

$$PID(s) = K_c \left( 1 + \frac{1}{T_i s} + s T_d \right)$$

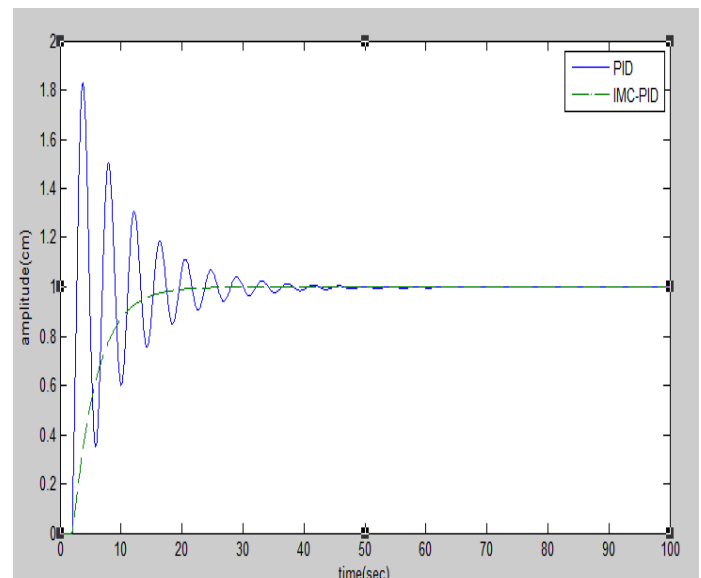
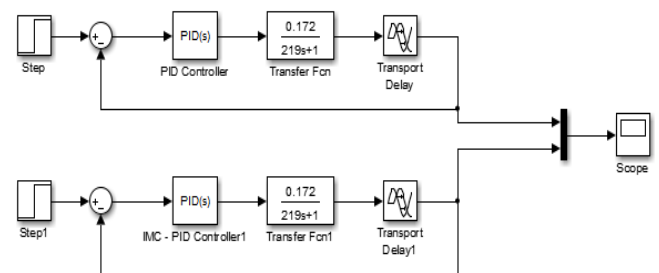
$K_c = \frac{\tau + \frac{\theta}{2}}{K(\lambda + \frac{\theta}{2})}$	$T_i = \frac{\theta}{2} + \tau$	$T_d = \frac{\frac{\theta}{2} + \tau}{2(\frac{\theta}{2} + \tau)}$
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model	Height	Transfer function	$G_c(s)$	$K_p$	$T_i$	$K_d$
1	10	$\frac{0.055}{74s+1} e^{-1s}$	$\frac{74s}{0.055s+1}$	269	3.63	0.5
2	15	$\frac{0.172}{219s+1} e^{-1s}$	$\frac{219s}{0.86s+1}$	254.6	1.16	0.48
3	20	$\frac{0.224}{349s+1} e^{-1s}$	$\frac{349s}{1.12s+1}$	311.6	0.89	0.5

### Simulation of IMC



### Comparison of the PID and IMC based PID



Response of PID and IMC-PID

**Hardware implementation kit****Comparison Table**

Set point (cm)	Controller	overshoot	Settling time (sec)
15	PID	18%	60
	IMC – PID	2.5%	25
20	PID	16%	38
	IMC -PID	1.8%	21

**6. CONCLUSION**

The nonlinearity of the conical tank is analyzed. Conventional PID, Cohen and coon tuned PID controller and IMC Tuned PID controller is implemented in simulation. Cohen and coon tuned PID Controller and IMC tuned PID Controller results are compared. IMC Tuned PID controller gives the better performance. As far as the tuning of the controller is concerned we have an optimum filter tuning factor  $\lambda$  (lambda) value which compromises the effects of discrepancies entering into the system to achieve the best performance. Thus, what we mean by the best filter structure is the filter that gives the best PID performance for the optimum  $\lambda$  value. The PID and IMC controllers are designed in such a way that the system is physically reliable. But due to the presence of dead time, the performance of the system is affected. The simulation results show the IMC based PID controller has minimum settling time and rise time in order to reach steady state value when compared to conventional controller. To avoid the minimum rise time advanced control schemes such as Model Reference Adaptive Controller will be implemented in future.

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